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WELL–POSEDNESS OF A DISSIPATIVE SYSTEM MODELING ELECTROHYDRODYNAMICS IN LEBESGUE SPACES

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Abstract. In this paper, we study a dissipative system of partial differential equations modeling the flow of electrohydrodynamics. This system consists of the Navier-Stokes equations with a source term coupled with the Nernst-Planck-Poisson equations for electronic charges. We establish local well-posedness of the initial value problem of this system in the critical and subcritical vector Lebesgue spaces. Moreover, we also prove that if the initial data is sufficiently small in critical Lebesgue spaces, then the solution is a global.

Mathematics subject classification (2010): 35K15, 35K55, 35Q35.
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Paper DEA-03-27

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