

## INDIRECT LOCALLY INTERNAL OBSERVABILITY AND CONTROLLABILITY OF WEAKLY COUPLED WAVE EQUATIONS

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*Abstract.* In this paper, we study the exact controllability of a system of weakly coupled wave equations with an internal locally control acted on only one equation. Using a piecewise multiplier method, we show that, for a sufficiently large time  $T$ , the observation of the velocity of the first component of the solution on a neighborhood of a part of the boundary allows us to get back a weakened energy of initial data of the second component of the solution, this if the coupling parameter is sufficiently small, but non vanishing. This result leads, by the HUM method, to prove that the total system is exactly controllable by means of one locally distributed control.

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