

## A CONCAVE-CONVEX QUASILINEAR ELLIPTIC PROBLEM SUBJECT TO A NONLINEAR BOUNDARY CONDITION

JOSÉ C. SABINA DE LIS

*Abstract.* This paper deals with the existence of a positive solution to the problem

$$\begin{cases} -\Delta_p u + u^{p-1} = u^{r-1}, & x \in \Omega, \\ |\nabla u|^{p-2} \frac{\partial u}{\partial \nu} = \lambda u^{q-1}, & x \in \partial\Omega, \end{cases}$$

where  $\Omega \subset \mathbb{R}^N$  is a bounded domain,  $\nu$  designates the unit outward normal to  $\partial\Omega$ ,  $\Delta_p$  is the  $p$ -Laplacian operator,  $1 < q < p < r \leq p^*$ ,  $p^* = Np/(N-p)$  if  $p < N$ ,  $p^* = \infty$  otherwise, while  $\lambda > 0$ . Our main result states the existence of  $\Lambda > 0$  so that positive solutions are only possible when  $0 < \lambda \leq \Lambda$  while the existence of a *minimal* positive solution is ensured in that range.

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