Differential
Equations
& Applications
Volume 3, Number 4 (2011), 487–502

ON ABSTRACT BARENBLATT EQUATIONS

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Abstract. In this paper we are interested in abstract problems of Barenblatt's type. In a first part, we investigate the problem $f(\partial_t u)+Au=g$ where f and A are maximal monotone operators and by assuming that A derives from a potential J. With general assumptions on the operators, we prove the existence of a solution. In the second part of the paper, we examine a stochastic version of the above problem: $f[\partial_t(u-\int_0^t h dw)]+Au=0$, with some restrictive assumptions on the data due principally to the framework of the Itô integral.

Mathematics subject classification (2010): 47J35, 35L90.
Keywords and phrases: abstract Barenblatt equation, monotone operators, stochastic equation.

REFERENCES

[1] R. A. ADAMS, *Sobolev spaces*, Pure and Applied Mathematics, 65, A Series of Monographs and Textbooks, New York, 1975.
[2] N. S. ADIMURTHI AND G. VALLET, *On the equation of Barenblatt-Sobolev*, Communications in Contemporary Mathematics, to appear.
[3] V. BARBU, *Nonlinear differential equations of monotone types in Banach spaces*, Springer Monographs in Mathematics, Springer, New York, 2010.
[4] G. I. BARENBLATT, *Similarity, self-similarity, and intermediate asymptotics*, New York, London: Consultants Bureau, XVII, 1979.
[5] C. BAUZET, J. GIACOMONI AND G. VALLET, *On a class of quasilinear Barenblatt equations*, Revista Real Academia de Ciencias de Zaragoza, to appear.
[6] J. M. BORWEIN AND J. D. VANDERWERFF, *Convex functions: constructions, characterizations and counterexamples*, Volume 109 of Encyclopedia of Mathematics and its Applications, Cambridge University Press, Cambridge, 2010.
[7] C. CHEN AND G. CHENG, *Anomalous dimension in the solution of the Barenblatt's equation*, J. Math. Phys., **39**, 3 (1998), 1589–1600.
[8] P. COLLI, *On some doubly nonlinear evolution equations in Banach spaces*, Japan J. Indust. Appl. Math., **9**, 2 (1992), 181–203.
[9] G. DÍAZ AND J. I. DÍAZ, *Finite extinction time for a class of nonlinear parabolic equations*, Comm. Partial Differential Equations, **4**, 11 (1979), 1213–1231.
[10] J. I. DÍAZ, J. S. LANGA AND J. VALERO, *On the asymptotic behaviour of solutions of a stochastic energy balance climate model*, Phys. D, **238**, 9-10 (2009), 880–887.
[11] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RUZICKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, Springer, 2011.
[12] K. S. HA, *Sur des semi-groupes non linéaires dans les espaces* $L^\infty(\Omega)$, J. Math. Soc. Japan, **31**, 4 (1979), 593–622.
[13] J. HULSHOF AND J.-L. VÁZQUEZ, *Self-similar solutions of the second kind for the modified porous medium equation*, European J. Appl. Math., **5**, 3 (1994), 391–403.
[14] S. KAMIN, L. A. PELETIER AND J. L. VÁZQUEZ, *On the Barenblatt equation of elastoplastic filtration*, Indiana Univ. Math. J., **40**, 4 (1991), 1333–1362.
[15] Y. KONISHI, *On the nonlinear semi-groups associated with* $u_t=\Delta\beta(u)$ *and* $\varphi(u_t)=\Delta u$, J. Math. Soc. Japan, **25**, (1973), 622–628.

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[16] N. IGBIDA, *Solutions auto-similaires pour une équation de Barenblatt*, Rev. Mat. Apl., **17**, 1 (1996), 21–36.
[17] C. PRÉVÔT C. AND M. RÖCKNER, *A concise course on stochastic partial differential equations*, Vol. 1905 of Lecture Notes in Mathematics, Springer, Berlin, 2007.
[18] G. SCHIMPERNA, A. SEGATTI AND U. STEFANELLI, *Well-posedness and long-time behavior for a class of doubly nonlinear equations*, Discrete Contin. Dyn. Syst., **18**, 1 (2007), 15–38.
[19] R. E. SHOWALTER, *Monotone operators in Banach space and nonlinear partial differential equations*, Vol. 49 of Mathematical Surveys and Monographs, American Mathematical Society, Providence RI, 1997.
[20] G. VALLET, *Stochastic perturbation of nonlinear degenerate parabolic problems*, Differential and integral equation, **21**, 11-12 (2008), 1055–1082.
[21] G. VALLET AND P. WITTBOLD, *On a stochastic first-order hyperbolic equation in a bounded domain*, Infinite Dimensional Analysis, QuantumProbability, **12**, 4 (2009), 1–39.