

WAVE EQUATION WITH $p(x,t)$ -LAPLACIAN AND DAMPING TERM: EXISTENCE AND BLOW-UP

STANISLAV ANTONTSEV

Abstract. In this work, we consider the Dirichlet problem for equation

$$u_{tt} = \operatorname{div}(a(x,t)|\nabla u|^{p(x,t)-2}\nabla u) + \alpha\Delta u + b(x,t)|u|^{\sigma(x,t)-2}u + f(x,t).$$

Under suitable conditions on the functions a , b , f , p , σ the local, global and blow up solutions have been discussed.

Mathematics subject classification (2010): 35B40, 35L70, 35L45.

Keywords and phrases: nonlinear wave equations, energy estimates, global existence, blow up, non-standard growth conditions.

REFERENCES

- [1] S. ANTONTSEV, *Wave equation with $p(x,t)$ -Laplacian and damping term: Existence and blow-up*, in Proceedings of the International Conference "Modern Problems of Applied Mathematics and Mechanics: Theory, Experiment and Applications" devoted to the 90th anniversary of Prof. N.N Yanenko, Novosibirsk, Russia, May 30-June 4, 2011.
- [2] S. ANTONTSEV, *Wave equation with $p(x,t)$ -Laplacian and damping term: Existence and blow-up*, in Abstracts, Nonlinear Models in Partial Differential Equations, An international congress on occasion of J.I. Diaz's 60th birthday, Toledo, Spain, June 14-17, 2011, p. 8.
- [3] S. ANTONTSEV, *Wave equation with $p(x,t)$ -Laplacian and damping term: Blow-up of solutions*, C. R. Mecanique, (2011, in press).
- [4] S. ANTONTSEV, M. CHIPOT, AND Y. XIE, *Uniqueness results for equations of the $p(x)$ -Laplacian type*, Adv. Math. Sci. Appl., **17** (2007), 287–304.
- [5] S. N. ANTONTSEV, J. I. DÍAZ, AND S. SHMAREV, *Energy Methods for Free Boundary Problems: Applications to Non-linear PDEs and Fluid Mechanics*, Birkhäuser, Boston, 2002. Progress in Nonlinear Differential Equations and Their Applications, Vol. 48.
- [6] S. N. ANTONTSEV AND J. F. RODRIGUES, *On stationary thermo-rheological viscous flows*, Ann. Univ. Ferrara, Sez.,VII. Sci. Mat., **52** (2006), 19–36.
- [7] S. N. ANTONTSEV AND S. I. SHMAREV, *Elliptic equations and systems with nonstandard growth conditions: existence, uniqueness and localization properties of solutions*, Journal Nonlinear Analysis, **65** (2006), 722–755.
- [8] S. N. ANTONTSEV AND S. I. SHMAREV, *Elliptic equations with anisotropic nonlinearity and non-standard growth conditions*, Elsevier, 2006. Handbook of Differential Equations. Stationary Partial Differential Equations, Elsevier, Vol. 3, Chapter 1, 1-100.
- [9] S. N. ANTONTSEV AND S. I. SHMAREV, *Parabolic equations with anisotropic nonstandard growth conditions*, in Internat. Ser. Numer. Math. 154, Birkhäuser, Verlag Basel/Switzerland, 2006, 33–44.
- [10] S. N. ANTONTSEV AND S. I. SHMAREV, *Extinction of solutions of parabolic equations with variable anisotropic nonlinearities*, Proceedings of the Steklov Institute of Mathematics, Moscow, Russia, **268** (2008), 2289–2301.
- [11] S. N. ANTONTSEV AND S. I. SHMAREV, *Anisotropic parabolic equations with variable nonlinearity*, Publ. Sec. Mat. Univ. Autònoma Barcelona, (2009), 355–399.
- [12] S. N. ANTONTSEV AND S. I. SHMAREV, *Localization of solutions of anisotropic parabolic equations*, Nonlinear Anal., **71** (2009), e725–e737.

- [13] S. N. ANTONTSEV AND S. I. SHMAREV, *Blow-up of solutions to parabolic equations with non-standard growth conditions*, Journal of Computational and Applied Mathematics, **234** (2010), 2633–2645.
- [14] G. AUTUORI, P. PUCCI, AND M. C. SALVATORI, *Asymptotic stability for anisotropic Kirchhoff systems*, J. Math. Anal. Appl., **352** (2009), 149–165.
- [15] G. AUTUORI, P. PUCCI, AND M. C. SALVATORI, *Global nonexistence for nonlinear Kirchhoff systems*, Arch. Rational Mech. Anal., **196** (2011), 489–516.
- [16] J. CLEMENTS, *Existence theorems for a quasilinear evolution equation*, SIAM J. Appl. Math., **26** (1974), 745–752.
- [17] L. DIENING, P. HARJULEHTO, P. HASTO, AND M. RÚŽIČKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, Springer, Berlin, 2011. Series: Lecture Notes in Mathematics, Vol. 2017, 1st Edition.
- [18] M. DREHER, *The wave equation for the p -Laplacian*, Hokkaido Math. J., **36** (2007), 21–52.
- [19] V. A. GALAKTIONOV AND S. I. POHOZAEV, *Blow-up and critical exponents for nonlinear hyperbolic equations*, Nonlinear Analysis, **53** (2003), 453–466.
- [20] H. GAO AND T. F. MA, *Global solutions for a nonlinear wave equation with p -Laplacian operator*, EJQTDE., (1999), 1–13.
- [21] V. GEORGIEV AND G. TODOROVA, *Existence of a solution of the wave equation with nonlinear damping and source terms*, J. Differential Equations, **109** (1994), 295–308.
- [22] J. HAEHNLE AND A. PROHL, *Approximation of nonlinear wave equations with nonstandard anisotropic growth conditions*, Math. Comp., **79** (2010), 189–208.
- [23] M. JAZAR AND R. KIWAN, *Blow-up results for some second-order hyperbolic inequalities with a nonlinear term with respect to the velocity*, J. Math. Anal. Appl., **327** (2007), 12–22.
- [24] M. JAZAR AND R. KIWAN, *Blow-up of a non-local semilinear parabolic equation with Neumann boundary conditions*, Ann. Inst. H. Poincaré Anal. Non Linéaire, **25** (2008), 215–218.
- [25] T. KATO, *Blow up of solutions of some nonlinear hyperbolic equations*, Manuscripta Math., **28** (1980), 235–268.
- [26] H. LEVINE AND G. TODOROVA, *Blow-up of solutions of the cauchy problem for a wave equations with nonlinear damping and source terms and positive initial energy*, in Proceedings of the American Mathematical Society, **129** (2000), 793–805.
- [27] J.-L. LIONS, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, 1969.
- [28] S. A. MESSAOUDI, *Blow up in a nonlinearly damped wave equation*, Math. Nachr., **231** (2001), 105–111.
- [29] S. A. MESSAOUDI, *Blow up in the Cauchy problem for a nonlinearly damped wave equation*, Commun. Appl. Anal., **7** (2003), 379–386.
- [30] S. A. MESSAOUDI, *On the decay of solutions for a class of quasilinear hyperbolic equations with non-linear damping and source terms*, Mathematical Methods in the Applied Sciences, **28** (2005), 1819–1828.
- [31] S. A. MESSAOUDI AND B. SAID HOUARI, *Global non-existence of solutions of a class of wave equations with non-linear damping and source terms*, Math. Methods Appl. Sci., **27** (2004), 1687–1696.
- [32] C. MU, R. ZENG, AND B. CHEN, *Blow-up phenomena for a doubly degenerate equation with positive initial energy*, Nonlinear Anal., **72** (2010), 782–793.
- [33] M. NAKAO AND Y. ZHIJIAN, *Global attractors for some quasi-linear wave equations with a strong dissipation*, Adv. Math. Sci. Appl., **17** (2007), 89–105.
- [34] J. P. PINASCO, *Blow-up for parabolic and hyperbolic problems with variable exponents*, Nonlinear Anal., **71** (2009), 1094–1099.
- [35] K. RAJAGOPAL AND M. RÚŽIČKA, *Mathematical modelling of electro-rheological fluids*, Cont. Mech. Therm., **13** (2001), 59–78.
- [36] M. RÚŽIČKA, *Electrorheological fluids: modeling and mathematical theory*, Springer, Berlin, 2000. Lecture Notes in Mathematics, 1748.
- [37] A. A. SAMARSKII, V. A. GALAKTIONOV, S. P. KURDYUMOV, AND A. P. MIKHAILOV, *Blow-up in quasilinear parabolic equations*, Walter de Gruyter & Co., Berlin, 1995. Translated from the 1987 Russian original by Michael Grinfeld and revised by the authors.
- [38] S. G. SAMKO, *Density $C_0^\infty(\mathbb{R}^n)$ in the generalized Sobolev spaces $W^{m,p(x)}(\mathbb{R}^n)$* , Dokl. Akad. Nauk, **369** (1999), 451–454.

- [39] J. SIMON, *Compact sets in the space $L^p(0,t;b)$* , Ann. Mat. Pura Appl., IV. Ser., **146** (1952), 65–96.
- [40] G. TODOROVA AND E. VITILLARIO, *Blow-up for nonlinear dissipative wave equations in \mathbb{R}^n* , J. Math. Anal. Appl., (2005), 242–257.
- [41] Z. WILSTEIN, *Global well-posedness for a nonlinear wave equation with p -Laplacian damping*, Dissertation, University of Nebraska-Lincoln, downloadable at :<http://digitalcommons.unl.edu/mathstudent/24>, (2011), 1–116.
- [42] Z. YANG, *Cauchy problem for quasi-linear wave equations with viscous damping*, J. Math. Anal. Appl., **320** (2006), 859–881.
- [43] Z. YANG AND G. CHEN, *Global existence of solutions for quasi-linear wave equations with viscous damping*, J. Math. Anal. Appl., **285** (2003), 604–618.
- [44] Y. ZHIJIAN, *Existence and asymptotic behaviour of solutions for a class of quasi-linear evolution equations with non-linear damping and source terms*, Mathematical Methods in the Applied Sciences, **25** (2002), 795–814.
- [45] Y. ZHIJIAN, *Global existence, asymptotic behavior and blowup of solutions for a class of nonlinear wave equations with dissipative term*, J. Differential Equations, **187** (2003), 520–540.
- [46] Y. ZHIJIAN, *Initial boundary value problem for a class of non-linear strongly damped wave equations*, Math. Methods Appl. Sci., **26** (2003), 1047–1066.
- [47] Y. ZHIJIAN, *Cauchy problem for a class of nonlinear dispersive wave equations arising in elastoplastic flow*, J. Math. Anal. Appl., **313** (2006), 197–217.
- [48] V. V. ZHIKOV, *On the density of smooth functions in Sobolev-Orlich spaces*, Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI), **310** (2004), 1–14.