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TRAJECTORY ATTRACTORS OF ENERGY BALANCE CLIMATE MODELS WITH BIO–FEEDBACK

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Abstract. Motivated by coupling an energy balance climate model and a two-species competition model for the bio-sphere, one is led to the study of functional reaction-diffusion equations with memory and a nonlocal Volterra operator. The existence of a trajectory attractor is established. The work is motivated by similar studies in [12] for a energy balance model with latent heat flux and uses techniques developed in [11] and [12]. It is a continuation of [18], where an abstract global existence and boundedness result was established.

Mathematics subject classification (2010): 35K65, 35K92, 37K57, 35R10, 45D99, 58J99.
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