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RECTIFIABLE OSCILLATIONS OF RADIALLY SYMMETRIC SOLUTIONS OF p–LAPLACE DIFFERENTIAL EQUATIONS

YŪKI NAITO, MERVAN PAŠIĆ, AND HIROYUKI USAMI

Abstract. Let $\Omega = \{x \in \mathbb{R}^N : r_0 \leqslant |x| < 1\}$ with $N \geqslant 2$ and $r_0 \in (0,1)$. We study a kind of geometric oscillatory and asymptotic behaviour near $|x| = 1$ of all radially symmetric solutions $u = u(x)$ of the p-Laplace partial differential equation (P): $-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = f(|x|)|u|^{p-2}u$ in Ω, $u = 0$ on $|x| = 1$ for $p > 1$. Necessary and sufficient conditions on the coefficient $f(|x|)$ are given such that $u(x)$ oscillates near $|x| = 1$ and the surface area of graph $\Gamma(u) \subseteq \mathbb{R}^{N+1}$ of $u(x)$ is finite-rectifiable oscillations, and infinite-nonrectifiable oscillations. The L^1-integrability and L^p-nonintegrability of $|\nabla u|$ on Ω for $p > 1$ are also considered.

Mathematics subject classification (2010): 26B15, 26B35, 34A26, 34C10, 34D05, 35B05, 35J25.
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