Differential
Equations
& Applications
Volume 4, Number 1 (2012), 67–94

EXISTENCE AND UNIQUENESS FOR DOUBLY NONLINEAR PARABOLIC EQUATIONS WITH NONSTANDARD GROWTH CONDITIONS

STANISLAV ANTONTSEV AND SERGEY SHMAREV

Abstract. We study the homogeneous Dirichlet problem for the equation

$$u_t = \sum_{i=1}^{n} D_i\big(a_i|D_i(|u|^{m(x)-1}u)|^{p_i(x,t)-2}D_i(|u|^{m(x)-1}u)\big) + b|u|^{\sigma(x,t)-2}u$$

with given exponents $m(x)$, $p_i(x,t)$ and $\sigma(x,t)$. It is proved that the problem has a solution in a suitable variable exponent Sobolev space. In dependence on the properties of the coefficient b and the exponents of nonlinearity, the solution exists globally or locally in time. The comparison principle and uniqueness are proved under additional restrictions on the data.

Mathematics subject classification (2010): 35K55, 35K65, 35K67, 35K92.
Keywords and phrases: nonlinear parabolic equations, double nonlinearity, nonstandard growth conditions.

REFERENCES

[1] S. ANTONTSEV, M. CHIPOT, AND S. SHMAREV, *Uniqueness and comparison theorems for solutions of doubly nonlinear parabolic equations with nonstandard growth conditions*, to appear in Commun. Pure Appl. Anal.

[2] S. N. ANTONTSEV, J. I. DÍAZ, AND S. SHMAREV, *Energy methods for free boundary problems. Applications to nonlinear PDEs and fluid mechanics*, Progress in Nonlinear Differential Equations and their Applications, **48**, Birkhäuser Boston, Inc., Boston, MA, 2002.

[3] S. N. ANTONTSEV AND S. I. SHMAREV, *Parabolic equations with anisotropic nonstandard growth conditions*, Free boundary problems, 33-44, Internat. Ser. Numer. Math., **154**, Birkhäuser, Basel, 2007.

[4] S. N. ANTONTSEV AND S. I. SHMAREV, *Anisotropic parabolic equations with variable nonlinearity*, Publ. Mat., **53**, 2 (2009), 355–399.

[5] S. N. ANTONTSEV AND S. I. SHMAREV, *Localization of solutions of anisotropic parabolic equations*, Nonlinear Anal., **71**, 12 (2009), 725–737.

[6] S. N. ANTONTSEV AND S. I. SHMAREV, *Blow-up of solutions to parabolic equations with nonstandard growth conditions*, J. Comput. Appl. Math., **234**, 9 (2010), 2633–2645.

[7] S. N. ANTONTSEV AND S. I. SHMAREV, *On the blow-up of solutions to anisotropic parabolic equations with variable nonlinearity*, Proc. Steklov Inst. Math., **270**, 1 (2010), 27–42.

[8] S. N. ANTONTSEV AND S. I. SHMAREV, *Vanishing solutions of anisotropic parabolic equations with variable nonlinearity*, J. Math. Anal. Appl., **361**, 2 (2010), 371–391.

[9] S. N. ANTONTSEV AND S. I. SHMAREV, *Parabolic equations with double variable nonlinearities*, Math. Comput. Simulation, **81**, 10 (2011), 2018–2032.

[10] S. N. ANTONTSEV AND S. I. SHMAREV, *Elliptic equations with triple variable nonlinearity*, Complex Var. Elliptic Equ., **56**, 7-9 (2011), 573–597.

[11] F. BERNIS, *Existence results for doubly nonlinear higher order parabolic equations on unbounded domains*, Math. Ann., **279**, 3 (1988), 373–394.

[12] C. CHEN, *Global existence and L^∞ estimates of solution for doubly nonlinear parabolic equation*, J. Math. Anal. Appl., **244**, 1 (2000), 133–146.

Paper DEA-04-05

[13] M. CHIPOT AND J.-F. RODRIGUES, *Comparison and stability of solutions to a class of quasilinear parabolic problems*, Proc. Roy. Soc. Edinburgh Sect. A, **110**, 3-4 (1988), 275–285.
[14] P. CIANCI, A. V. MARTYNENKO, AND A. F. TEDEEV, *The blow-up phenomenon for degenerate parabolic equations with variable coefficients and nonlinear source*, Nonlinear Anal., **73**, 7 (2010), 2310–2323.
[15] S. P. DEGTYAREV AND A. F. TEDEEV, *Estimates for the solution of the Cauchy problem with increasing initial data for a parabolic equation with anisotropic degeneration and double nonlinearity*, Dokl. Math., **76**, 3 (2007), 824–827.
[16] J. DÍAZ AND J. PADIAL, *Uniqueness and existence of a solution in $BV_t(q)$ space to a doubly nonlinear parabolic problem*, Publ. Mat., **40**, 2 (1996), 527–560.
[17] J. DÍAZ AND F. THÉLIN, *On a nonlinear parabolic problem arising in some models related to turbulent flows*, SIAM J. Math. Anal., **25**, 4 (1994), 1085–1111.
[18] L. DIENING, P. HARJULEHTO, P. HÄSTÖ, AND M. RUŽIČKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, Springer, Berlin, 2011.
[19] D. E. EDMUNDS AND J. RÁKOSNÍK, *Density of smooth functions in $W^{k,p(x)}(\Omega)$*, Proc. Roy. Soc. London Ser. A, **437** (1992), 229–236.
[20] K. ISHIGE, *On the existence of solutions of the Cauchy problem for a doubly nonlinear parabolic equation*, SIAM J. Math. Anal., **27**, 5 (1996), 1235–1260.
[21] P. A. HÄSTÖ, *On the density of continuous functions in variable exponent Sobolev space*, Rev. Mat. Iberoam., **23**, 1 (2007), 213–234.
[22] G. I. LAPTEV, *Solvability of second-order quasilinear parabolic equations with double degeneration*, Siberian Math. J., **38**, 6 (1997), 1160–1177.
[23] J.-L. LIONS, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, 1969.
[24] S. SAMKO, *On a progress in the theory of Lebesgue spaces with variable exponent: maximal and singular operators*, Integral Transforms Spec. Funct., **16**, 5-6 (2005), 461–482.
[25] M. SANGO, *On a doubly degenerate quasilinear anisotropic parabolic equation*, Analysis (Munich), **23**, 3 (2003), 249–260.
[26] J. SIMON, *Compact sets in the space $L^p(0,t;B)$*, Ann. Mat. Pura Appl., **146**, 4 (1987), 65–96.
[27] A. F. TEDEEV, *The interface blow-up phenomenon and local estimates for doubly degenerate parabolic equations*, Appl. Anal., **86**, 6 (2007), 755–782.
[28] V. V. ZHIKOV, *On the density of smooth functions in Sobolev-Orlicz spaces*, J. Math. Sci. (N. Y.), **132**, 3 (2006), 285–294.
[29] V. V. ZHIKOV AND S. E. PASTUKHOVA, *Lemmas on compensated compactness in elliptic and parabolic equations*, Proc. Steklov Inst. Math., **270**, 1 (2010), 104–131.