

A QUASILINEAR PARABOLIC MODEL FOR POPULATION EVOLUTION

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Abstract. A quasilinear parabolic problem is investigated. It models the evolution of a single population species with a nonlinear diffusion and a logistic reaction function. We present a new treatment combining standard theory of monotone operators in $L^2(\Omega)$ with some order-preserving properties of the evolutionary equation. The advantage of our approach is that we are able to obtain the *existence* and *long-time asymptotic behavior* of a weak solution almost simultaneously. We do not employ any uniqueness results; we rely on the uniqueness of the minimal and maximal solutions instead. At last, we answer the question of (long-time) survival of the population in terms of a critical value of a spectral parameter.

Mathematics subject classification (2010): 35B40, 35K59, 35K92, 92D25.

Keywords and phrases: quasilinear parabolic problem, strongly monotone operator, monotone iterations, sub- and supersolutions, asymptotic behavior, logistic equation, p -Laplacian.

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