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A NONLINEAR PARABOLIC–HYPERBOLIC SYSTEM FOR CONTACT INHIBITION OF CELL–GROWTH

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Abstract. We consider a tumor growth model involving a nonlinear system of partial differential equations which describes the growth of two types of cell population densities with contact inhibition. In one space dimension, it is known that global solutions exist and that they satisfy the so-called *segregation property*: if the two populations are initially segregated - in mathematical terms this translates into disjoint supports of their densities - this property remains true at all later times. We apply recent results on transport equations and regular Lagrangian flows to obtain similar results in the case of arbitrary space dimension.

Mathematics subject classification (2010): 35R35, 35M20, 35Q80, 92C17, 92C50.
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