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# EXISTENCE OF POSITIVE SOLUTIONS FOR SECOND ORDER MULTI–POINT BOUNDARY VALUE PROBLEMS

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*Abstract.* We study the second order nonlinear differential equation:

$$(E) \quad y''+h(t)f(y)=0,\ 0<t<1,\ h(t)\in L^1(0,1),\ f(y)\in C(\mathbb{R},\mathbb{R}_+)$$

subject to multi-point boundary conditions:

(i) $y(0)=\sum_{i=1}^{m}\alpha_i y(\xi_i),\ y(1)=\sum_{i=1}^{m}\beta_i y(\xi_i),$

(ii) $y'(0)=\sum_{i=1}^{m}\alpha_i y'(\xi_i),\ y(1)=\sum_{i=1}^{m}\beta_i y(\xi_i),$

(iii) $y(0)=\sum_{i=1}^{m}\alpha_i y(\xi_i),\ y'(1)=\sum_{i=1}^{m}\beta_i y'(\xi_i),$

where $0<\xi_1<\cdots<\xi_m<1, \alpha_i\geqslant 0, \beta_i\geqslant 0, i=1,2,\cdots,m$. We also assume that $h(t)$ is non-negative and can be singular at $t=0$ or $t=1$ or both and $\alpha_1+\cdots+\alpha_m<1$, $\beta_1+\cdots+\beta_m<1$. We prove existence theorems for positive solutions of (E) (i), (E) (ii) and (E) (iii) when the limit $f(y)/y$ as $y\to 0$ and $y\to\infty$ do not necessarily exist. Our results extend recent results of Zhang and Sun [18] for the boundary value problem (E)(i) when $\alpha_i\equiv 0$ for all $i=1,\ldots,m$.

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