

DETAILED STUDY OF BIFURCATIONS IN AN EPIDEMIC MODEL ON A DYNAMIC NETWORK

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Abstract. The bifurcations in a four-variable ODE model of an SIS type epidemic on an adaptive network are studied. The model describes the propagation of the epidemic on a network where links (or edges) of different type (i.e. SI , II and SS) can be activated or deleted according to a simple rule consisting of random link activation and deletion. In the case when II links cannot be neither deleted nor created it is proved that the system can have at most three steady states with the trivial, disease-free steady state being one of them. It is shown that a stable endemic steady state can appear through a transcritical bifurcation, or a stable and an unstable endemic steady state arise as a result of saddle-node bifurcation. Moreover, at the endemic steady state a Hopf bifurcation may occur giving rise to stable oscillation. The bifurcation curves in the parameter space are determined analytically using the parametric representation method. For certain parameter regimes or bifurcation types, analytical results based on the ODE model show good agreement when compared to results based on individual-based network simulations. When agreement between the two modelling approaches holds, the ODE-based model provides a faster and more reliable tool that can be used to explore full spectrum of model behaviour.

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