

OSCILLATION AND NONOSCILLATION CRITERIA FOR EVEN ORDER NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. This paper is devoted to the study of the oscillatory and nonoscillatory behavior of even order nonlinear functional differential equations with deviating argument of the type

$$(p(t)|x^{(n)}(t)|^\alpha \operatorname{sgn} x^{(n)}(t))^{(n)} + q(t)|x(g(t))|^\beta \operatorname{sgn} x(g(t)) = 0. \quad (A_g)$$

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