

QUASILINEAR ELLIPTIC PROBLEMS WITH CRITICAL EXPONENTS AND DISCONTINUOUS NONLINEARITIES

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Abstract. Using a recent fixed point theorem in ordered Banach spaces by S. Carl and S. Heikkilä, we study the existence of weak solutions to nonlinear elliptic problems $-\operatorname{div}_x(x, \nabla u) = f(x, u)$ in a bounded domain $\Omega \subset \mathbb{R}^n$ with Dirichlet boundary condition. In particular, we prove that for some suitable function g , which may be discontinuous, and δ small enough, the p -Laplace equation

$$-\operatorname{div}(|\nabla u|^{p-2} \nabla u) = |u|^{p^*-2} u + \delta g(x, u)$$

has a positive solution which goes to 0 as $\delta \rightarrow 0^+$, where p^* is the critical exponent.

Mathematics subject classification (2010): 35J62, 35D30, 47H10.

Keywords and phrases: p -Laplace, critical exponents, discontinuous nonlinearities, positive solutions.

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