

EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR THE NONLINEAR KLEIN–GORDON EQUATION COUPLED WITH BORN–INFELD THEORY ON BOUNDED DOMAIN

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Abstract. In this paper, we prove some existence and multiple results for the following nonlinear Klein-Gordon equation coupled with Born-Infeld theory

$$\begin{cases} \Delta u = (m^2 - (\omega + \phi)^2)u - f(x, u), & \text{in } \Omega, \\ \Delta \phi + \beta \Delta_4 \phi = 4\pi(\omega + \phi)u^2, & \text{in } \Omega, \\ u = \phi = 0, & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 3$) is a bounded domain with smooth boundary, and $f \in C(\bar{\Omega}, \mathbb{R})$ satisfies some assumptions.

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