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EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR THE NONLINEAR KLEIN–GORDON EQUATION COUPLED WITH BORN–INFELD THEORY ON BOUNDED DOMAIN

KAIMIN TENG

Abstract. In this paper, we prove some existence and multiple results for the following nonlinear Klein-Gordon equation coupled with Born-Infeld theory

$$\begin{cases} \Delta u = \big(m^2 - (\omega+\phi)^2\big)u - f(x,u), & \text{in } \Omega, \\ \Delta\phi + \beta\Delta_4\phi = 4\pi(\omega+\phi)u^2, & \text{in } \Omega, \\ u = \phi = 0, & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geqslant 3)$ is a bounded domain with smooth boundary, and $f \in C(\overline{\Omega}, \mathbb{R})$ satisfies some assumptions.

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