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EXISTENCE RESULTS FOR SECOND ORDER THREE–POINT BOUNDARY VALUE PROBLEMS

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Abstract. The paper is devoted to the study of second order differential equations and systems with nonlinear three point boundary conditions. The existence of solutions is proved using fixed point theorems: Banach's and Boyd-Wong's contraction principles, Perov's and Schauder's fixed point theorems.

Mathematics subject classification (2010): 34B10, 34B15.
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