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EXISTENCE OF POSITIVE SOLUTIONS FOR A SINGULAR PROBLEMS OF CAFFARELLI–KOHN–NIRENBERG–LIN TYPE

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Abstract. In this paper, we study the existence of nontrivial critical points of the functional

$$J_{\lambda,\mu}(u,v) = \frac{1}{p}\int_{\mathbb{R}^N}(\big||x|^{-a}\nabla^k u\big|^p - \lambda f(x)\big||x|^{-(a+k)}u\big|^p)dx + \frac{1}{q}\int_{\mathbb{R}^N}(\big||x|^{-a}\nabla^k v\big|^q - \mu g(x)\big||x|^{-(a+k)}v\big|^q)dx - \int_{\mathbb{R}^N} h(x)|x|^{-b(\alpha+\beta+2)}|u|^{\alpha+1}|v|^{\beta+1}dx,$$

related to the Caffarelli-Kohn-Nirenberg inequality and its higher order variant by Lin.

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