

## EXISTENCE OF SOLUTIONS FOR A SINGULAR FOURTH-ORDER $p$ -LAPLACIAN SYSTEM

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*Abstract.* This work is devoted to proving existence of solutions for a singular fourth-order  $p$ -Laplacian system. The nonlinearities depend on the solution and first derivatives and may exhibit singularities. Existence results are proved using the Leray-Schauder nonlinear alternative. Examples of applications illustrate each one of the obtained results.

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