

RIEMANN PROBLEM FOR ONE-DIMENSIONAL SYSTEM OF CONSERVATION LAWS OF MASS, MOMENTUM AND ENERGY IN ZERO-PRESSURE GAS DYNAMICS

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Abstract. This paper studies the one-dimensional Riemann problem for the system of conservation laws of mass, momentum and energy in zero-pressure gas dynamics. Using the characteristic analysis method, two kinds of solutions are obtained: vacuum and delta-shock solution. Under suitable generalized Rankine-Hugoniot relation and entropy condition, both existence and uniqueness of delta-shock solutions are established. These analytical results well match the results obtained through numerical simulations.

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