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GLOBAL CURVE OF POSITIVE SOLUTIONS FOR $\varphi-$ LAPLACIAN DIRICHLET BVP WITH AT MOST ONE TURNING POINT

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Abstract. Under suitable conditions we prove that the set of positive solutions to the $\varphi-$Laplacian boundary value problem

$$-(\varphi(u'))' = \lambda f(u) \text{ in } (0,1);\ u(0) = u(1) = 0,$$

where $\lambda > 0$ is a real parameter, φ is an odd increasing homeomorphism of $\mathbb{R}$ and $f \in C([0,+\infty),[0,+\infty))$, consists on a curve $\|u\| \to \lambda(\|u\|)$.

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