

POSITIVE PSEUDO-SYMMETRIC SOLUTIONS FOR A NONLOCAL p -LAPLACIAN BOUNDARY VALUE PROBLEM

L. X. TRUONG, P. D. PHUNG AND B. T. QUAN

Abstract. This paper is devoted to the study of the following nonlocal p -Laplacian functional differential equation

$$-(\phi_p(x'(t)))' = \frac{\lambda f(t, x(t), x'(t))}{\left(\int_0^1 f(s, x(s), x'(s)) ds\right)^n}, \quad 0 < t < 1,$$

subject to multi point boundary conditions. We obtain some results on the existence of at least one (when $n \in \mathbb{Z}^+$) or triple (when $n = 0$) pseudo-symmetric positive solutions by using fixed-point theory in cone.

Mathematics subject classification (2010): 34B07, 34B10, 34B18, 34B27.

Keywords and phrases: boundary value problem, pseudo-symmetric solutions, p -Laplacian, Leggett-Williams fixed point theorem, Guo-Krasnoselskii fixed point theorem.

REFERENCES

- [1] M. R. S. AMMI, D. F. M. TORRES, *Existence of positive solutions for nonlocal p -Laplacian thermistor problems on time scales*, J. Inequalities in Pure and Appl. Math., **8** (2007), 1–10.
- [2] R.I. AVERY, J. HENDERSON, *Three symmetric positive solutions for a second order boundary value problem*, Appl. Math. Lett., **13** (2000), 1–7.
- [3] R. AVERY, J. HENDERSON, *Existence of three positive pseudo-symmetric solutions for a one dimensional p -Laplacian operator*, J. Math. Anal. Appl., **277** (2003), 395–404.
- [4] R.P. AGARWAL, H. L'U, D. O'REGAN, *Eigenvalues and the one-dimensional p -Laplacian*, J. Math. Anal. Appl., **266** (2002), 383–340.
- [5] Y. GUO, W. GE, *Three positive solutions for the one-dimensional p -Laplacian*, J. Math. Anal. Appl., **286** (2003), 491–508.
- [6] Y. GUO, W. GE, *Positive solutions for three point boundary value problems with dependence on the first order derivative*, J. Math. Anal. Appl., **290** (2004), 291–301.
- [7] V.A. IL'IN, E.I. MOISEEV, *Nonlocal boundary value problem of the first kind for a Sturm-Liouville operator*, Differ. Equations, **23** (1987), 979–987.
- [8] V.A. IL'IN, E.I. MOISEEV, *Nonlocal boundary value problem of the first kind for the Sturm-Liouville operator in the differential and difference treatment*, Differ. Equations, **23** (1987), 1198–1207.
- [9] M. A. KRASNOSEL'SKII, *Positive solution of Operator Equations*, Noordho, Groningen, 1964.
- [10] R.W. LEGGETT, L.R. WILLIAMS, *Multiple positive fixed points of nonlinear operators on ordered Banach spaces*, Indiana Univ. Math. J., **28** (1979), 673–688.
- [11] D. MA, W. GE, *Existence and iteration of positive pseudo-symmetric solutions for a three-point second-order p -Laplacian BVP*, Appl. Math. Lett., **20** (2007), 1244–1249.
- [12] B. SUN, W. GE, *Successive iteration and positive pseudo-symmetric solutions for a three-point second-order p -Laplacian boundary value problems*, Appl. Math. Comput., **188** (2007), 1772–1779.

- [13] Y. H. SU, W. WU, X. YANG, *Existence Theory for Pseudo-Symmetric Solution to p -Laplacian Differential Equations Involving Derivative*, *Abstract and Applied Analysis*, **2011** (2011), Article ID 182831, 19 pages.
- [14] L. X. TRUONG, L. T. P. NGOC, N. T. LONG, *Positive solutions for an m -point boundary value problem*, *Electron. J. Differential Eqns.*, **111** (2008), 1–11.
- [15] L.X. TRUONG, P.D. PHUNG, *Existence of positive solutions for a multi-point four-order boundary-value problem*, *Electron. J. Differential Eqns.*, **119** (2011), 1–10.
- [16] J.Y. WANG, *The existence of positive solutions for the one-dimensional p -laplacian*, *Proc. Amer. Math. Soc.*, **125** (1997), 2275–2283.
- [17] Y. YANG, *Existence of positive pseudo-symmetric solutions for one dimensional p -Laplacian boundary value problem*, *Electron. J. Differential Eqns.*, **70** (2007), 1–6.