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SYSTEMS OF ELLIPTIC EQUATIONS INVOLVING MULTIPLE INVERSE–SQUARE POTENTIALS AND CRITICAL EXPONENTS

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Abstract. In this paper, a system of elliptic equations is investigated, which involves multiple critical Sobolev exponents and singular points. The best Sobelev constant related to the system is studied, which is verified to be independent of the location of singular points. By a variant of the concentration compactness principle and the mountain-pass argument, the existence of positive solutions to the system is proved. At last, the existence of sign-changing solutions to the system is also established on the basis of the mountain-pass-type positive solutions.

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