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QUASILINEAR ELLIPTIC PROBLEM WITH HARDY POTENTIAL AND A REACTION–ABSORBTION TERM

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Abstract. We consider the following quasilinear elliptic problem

$$\begin{cases} -\Delta_p u \pm u^q = \lambda \dfrac{u^{p-1}}{|x|^p} + h & \text{in } \Omega, \\ u \geqslant 0 \text{ and } u = 0 & \text{on } \partial\Omega, \end{cases}$$

where, $1 < p < N, \Omega \subset \mathbb{R}^N$ is a bounded regular domain such that $0 \in \Omega$, $q > p-1$ and h is a nonnegative measurable function with suitable hypotheses. The main goal of this paper is to analyze the interaction between the Hardy potential, and the term u^q, in order to get existence and non existence of positive solution. We can summarize our main results, in the two following points:

(i) If u^q appears as a reaction term, then we show the existence of a critical exponent $q_+(\lambda)$, such that for $q > q_+$, the considered problem has no positive distributional solution. If $q < q_+$ we find solutions under suitable hypothesis on h.

(ii) If u^q appears as an absorption term, then there exists q_* such that if $q > q_*$, the problem under consideration has a positive solution for all $\lambda > 0$ and for all $h \in L^1(\Omega)$. The optimality of q_* is proved in the sense that if $q < q_*$, then nonexistence holds if $\lambda > \Lambda_{N,p}$.

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