

HYBRID FIXED POINT THEORY IN PARTIALLY ORDERED NORMED LINEAR SPACES AND APPLICATIONS TO FRACTIONAL INTEGRAL EQUATIONS

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Abstract. In this paper, some basic hybrid fixed point theorems of Banach and Schauder type and some hybrid fixed point theorems of Krasnoselskii type involving the sum of two operators are proved in a partially ordered normed linear spaces which are further applied to nonlinear Volterra fractional integral equations for proving the existence of solutions under certain monotonic conditions blending with the existence of either a lower or an upper solution type function.

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