

## PERTURBATION RESULTS FOR SOME NONLINEAR EQUATIONS INVOLVING FRACTIONAL OPERATORS

SIMONE SECCHI

*Abstract.* By using a perturbation technique in critical point theory, we prove the existence of solutions for two types of nonlinear equations involving fractional differential operators.

*Mathematics subject classification* (2010): 35Q55, 35A15, 35J20.

*Keywords and phrases:* Perturbation methods, pseudo-relativistic Hartree equation, fractional laplacian.

### REFERENCES

- [1] A. AMBROSETTI, M. BADIÀLE, AND S. CINGOLANI, *Semiclassical states of nonlinear Schrödinger equations*, Arch. Rational Mech. Anal. **140**, No.3 (1997), 285–300.
- [2] A. AMBROSETTI, A. MALCHIODI, *Perturbation methods and semilinear elliptic problems on  $\mathbb{R}^n$* , volume 240 of *Progress in Mathematics*, Birkhäuser Verlag, Basel, 2006.
- [3] A. AMBROSETTI, A. MALCHIODI, AND S. SECCHI, *Multiplicity results for some nonlinear Schrödinger equations with potentials*, Arch. Ration. Mech. Anal. **159**, No.3 (2001), 253–271.
- [4] M. BERTI, P. BOLLE, *Homoclinics and chaotic behavior for perturbed second order systems*, Annali di Matematica Pura ed applicata, **CLXXVI** (1999), 323–378.
- [5] L. CAFFARELLI, L. SILVESTRE, *An extension problem related to the fractional Laplacian*, Comm. Partial Differential Equations, **32** (2007), 1245–1260.
- [6] S. CINGOLANI, S. SECCHI AND M. SQUASSINA, *Semi-classical limit for Schrödinger equations with magnetic field and Hartree-type nonlinearities*, Proc. Roy. Soc. Edinburgh Sect. A, **140** No.5 (2010), 973–1009.
- [7] V. COTI ZELATI, M. NOLASCO, *Existence of ground states for nonlinear, pseudorelativistic Schrödinger equations*, Rend. Lincei Mat. Appl., **22** (2011), 51–72.
- [8] V. COTI ZELATI, M. NOLASCO, *Ground states for pseudo-relativistic Hartree equations of critical type*, Preprint 2012.
- [9] E. DI NEZZA, G. PALATUCCI AND E. VALDINOCI, *Hitchhiker’s guide to the fractional Sobolev spaces*, Bull. Sci. Math., **136**, No.5 (2012), 521–573.
- [10] S. DIPIERRO, G. PALATUCCI AND E. VALDINOCI, *Existence and symmetry results for a Schrödinger type problem involving the fractional laplacian*, Le Matematiche, **68**, n.1 (2013).
- [11] P. FELMER, A. QUAAS AND J. TAN, *Positive solutions of the nonlinear Schrödinger equation with the fractional laplacian*, To appear on Proc. Roy. Soc. Edinburgh Sect. A.
- [12] R.L. FRANK, *On the uniqueness of ground states of non-local equations*, Journées Équations aux Dérivées Partielles 2011, Exp. No. V.
- [13] R.L. FRANK AND E. LENZMANN, *Uniqueness and nondegeneracy of ground states for  $(-\Delta)^s Q + Q - Q^{\alpha+1} = 0$  in  $\mathbb{R}$* , To appear in Annals of Mathematics.
- [14] R.L. FRANK, E. LENZMANN AND L. SILVESTRE, *Uniqueness of radial solutions for the fractional Laplacian*, Preprint, 2013, <http://arxiv.org/abs/1302.2652>.
- [15] E. LENZMANN, *Uniqueness of ground states for pseudorelativistic Hartree equations*, Analysis and PDE **2**, No.1, 2009.
- [16] Y.Y. LI, *On a singularly perturbed elliptic equation*, Adv. Differential Equations, **2**, No.6 (1997), 955–980.

- [17] E.H. LIEB, H.-T. YAOU, *The Chandrasekhar theory of stellar collapse as the limit of quantum mechanics*, *Comm. Math. Phys.*, **112** (1987).
- [18] D. MUGNAI, *The pseudorelativistic Hartree equation with a general nonlinearity: existence, non existence and variational identities*, Preprint 2012.
- [19] S. SECCHI, *Ground state solutions for nonlinear fractional Schrödinger equations in  $\mathbb{R}^n$* , *J. Math. Phys.*, in press.
- [20] J. TAN, Y. WANG AND J. YANG, *Nonlinear fractional field equations*, *Nonlinear analysis*, **72** (2012), 2098–2110.