

POSITIVE SOLUTION OF CRITICAL HARDY-SOBOLEV ELLIPTIC SYSTEMS WITH THE BOUNDARY SINGULARITY

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Abstract. In this paper, we are concerned with the existence of positive solutions to the system

$$\begin{cases} -\Delta u = \frac{2p}{p+q} u^{p-1} v^q + \frac{2\lambda\alpha}{\alpha+\beta} \frac{u^{\alpha-1} v^\beta}{|x|^s}, & \text{in } \Omega, \\ -\Delta v = \frac{2q}{p+q} u^p v^{q-1} + \frac{2\lambda\beta}{\alpha+\beta} \frac{u^\alpha v^{\beta-1}}{|x|^s}, & \text{in } \Omega, \\ u > 0, v > 0, & \text{in } \Omega, \\ u = v = 0, & \text{on } \partial\Omega, \end{cases} \quad (0.1)$$

where Ω is a C^2 domain in \mathbb{R}^N with $0 \in \partial\Omega$, $0 < s < 2$, $\lambda > 0$, $p+q = 2^* = \frac{2N}{N-2}$, $\alpha+\beta = 2^*(s) = \frac{2(N-s)}{N-2}$, $N \geq 3$. We show that if $\Omega = \mathbb{R}_+^N$, problem (0.1) possesses a least energy solution and if Ω is bounded, $0 \in \partial\Omega$, there exists $\lambda^* > 0$ such that problem (0.1) has at least a positive solution provided $0 < \lambda < \lambda^*$.

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