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SINGLE POINT BLOW–UP SOLUTIONS TO THE HEAT EQUATION WITH NONLINEAR BOUNDARY CONDITIONS

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Abstract. We study finite blow-up solutions of the heat equation with nonlinear boundary conditions. We provide a sufficient condition for the single point blow-up at the origin and a precise spacial singularity of the blow-up profile.

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Paper DEA-05-17

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