GLOBAL EXISTENCE AND DECAY ESTIMATES FOR NONLINEAR KIRCHHOFF–TYPE EQUATION WITH BOUNDARY DISSIPATION

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Abstract. In this paper, we consider the initial-boundary value problem for nonlinear Kirchhoff-type equation

$$u_{tt} - \varphi(\|\nabla u\|_2^2)\Delta u - a\Delta u_t = b|u|^{\beta-2}u,$$

where $a, b > 0$ and $\beta > 2$ are constants, φ is a C^1-function such that $\varphi(s) \geqslant \lambda_0 > 0$ for all $s \geqslant 0$. Under suitable conditions on the initial data, we show the existence and uniqueness of global solution by means of the Galerkin method and the uniform decay rate of the energy by an integral inequality.

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