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OSCILLATION OF DELAY DYNAMIC EQUATIONS WITH OSCILLATING COEFFICIENTS

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Abstract. In this paper, we study the following delay dynamic equation

$$x^{\Delta}(t)+p(t)x(\tau(t))=0 \text{ for } t\in[t_0,\infty)_{\mathbb{T}},$$

where $t_0\in\mathbb{T}$, $\sup\mathbb{T}=\infty$, $p\in C_{rd}([t_0,\infty)_{\mathbb{T}},\mathbb{R})$ alternates in sign infinitely many times and $\tau\in C_{rd}([t_0,\infty)_{\mathbb{T}},\mathbb{T})$ is a strictly increasing unbounded function satisfying $\tau(t)\leqslant t$ for all $t\in[t_0,\infty)_{\mathbb{T}}$. Our results extend recent results for arbitrary time scales.

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