SUFFICIENT CONDITIONS FOR EXISTENCE OF SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS WITH NONLOCAL BOUNDARY CONDITIONS

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Abstract. We provide easily verifiable sufficient conditions for the existence of solutions to nonlinear ordinary differential equations subject to nonlocal boundary conditions. These conditions are based on the solution space of the corresponding linear, homogeneous problem and on the size of the nonlinear perturbation. The results presented here are more general nonlinearities than those in [17] and [26].

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