ANALYTIC SMOOTHING EFFECT FOR A SYSTEM
OF NONLINEAR SCHRÖDINGER EQUATIONS

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Abstract. We prove the global existence of analytic solutions to the Cauchy problem for a system of nonlinear Schrödinger equations with quadratic interaction in space dimension $n \geqslant 3$ under the mass resonance condition. Lagrangian formulation is also described.

REFERENCES

[1] J.-L. BONA, Z. GRUJIĆ AND H. KALISCH, *Global solutions of the derivative Schrödinger equation in a class of functions analytic in a strip*, J. Differential Equations, **229** (2006), 186–203, doi:10.1016/j.jde.2006.04.013.

[2] T. CAZENAVE, *Semilinear Schrödinger equations*, Courant Lecture Notes in Math., **10** Amer. Math. Soc., 2003.

[3] T. CAZENAVE, AND F.B. WEISSLER, *The Cauchy problem for the critical nonlinear Schrödinger equation in H^s*, Nonlinear Analysis, TMA., **14** (1990), 807–836, doi:10.1016/0362-546X(90)90023-A.

[4] A. DEBOUARD, *Analytic solution to nonelliptic nonlinear Schrödinger equations*, J. Differential Equations, **104** (1993), 196–213, doi:10.1006/jdeq.1993.1069.

[5] J. GINIBRE, *Introduction aux équations de Schrödinger non linéaires*, Paris Onze Edition, L161. Université Paris-Sud, 1998.

[6] N. HAYASHI, *Global existence of small analytic solutions to nonlinear Schrödinger equations*, Duke Math. J., **60** (1990), 717–727, doi:10.1215/S0012-7094-90-06029-6.

[7] N. HAYASHI AND K. KATO, *Analyticity in time and smoothing effect of solutions to nonlinear Schrödinger equations*, Commun. Math Phys., **184** (1997), 273–300, doi:10.1007/s002200050061.

[8] N. HAYASHI, C. LI AND T. OZAWA, *Small data scattering for a system of nonlinear Schrödinger equations*, Differ. Equ. Appl., **3** (2011), 415–426.

[9] N. HAYASHI AND T. OZAWA, *On the derivative nonlinear Schrödinger equation*, Physica D, **55** (1992), 14–36, doi:10.1016/0167-2789(92)90185-P.

[10] N. HAYASHI, T. OZAWA AND K. TANAKA, *On a system of nonlinear Schrödinger equations with quadratic interaction*, Ann. Inst. H. Poincaré Anal. Non Linéaire, **30**, (2013), 661–690, doi:10.1016/j.anihpc.2012.10.007.

[11] N. HAYASHI AND S. SAITOH, *Analyticity and smoothing effect for the Schrödinger equation*, Ann. Inst. Henri Poincaré, Phys. Théor., **52** (1990), 163–173.

[12] N. HAYASHI AND S. SAITOH, *Analyticity and global existence of small solutions to some nonlinear Schrödinger equations*, Commun. Math. Phys., **129**,(1990) 27–41, doi:10.1007/BF02096777.

[13] G. HOSHINO AND T. OZAWA, *Analyticity smoothing effect for nonlinear Schrödinger equation in two space dimensions*, Osaka J. Math, (in press).

[14] K. KATO AND K. TANIGUCHI, *Gevrey regularizing effect for nonlinear Schrödinger equations*, Osaka. J. Math., **33** (1996), 863–880.

[15] T. KATO, *On nonlinear Schrödinger equations.II H^s-solutions and unconditional well-posedness*, J. d'Anal. Math., **67** (1995), 281–306.

[16] T. KATO AND K. MASUDA, *Nonlinear evolution equations and analyticity*, Ann. Inst. Henri Poincaré, Analyse non linéaire, **3** (1986), 455–467.

[17] K. NAKAMITSU, *Analytic finite energy solutions of the nonlinear Schrödinger equation*, Commun. Math. Phys., **260** (2005), 117–130, doi:10.1007/s00220-005-1405-7.

[18] M. NAKAMURA AND T. OZAWA, *Low energy scattering for nonlinear Schrödinger equations in fractional order Sobolev spaces*, Rev. Math. Phys., **9** (1997), 397–410, doi:10.1142/S0129055X97000154.

[19] K. NAKANISHI, *Asymptotically-free solutions for the short-range nonlinear Schrödinger equation*, SIAM J.Math.Anal., **32** (2001), 1265–1271, doi:10.1137/S0036141000369083.

[20] K. NAKANISHI AND T. OZAWA, *Remarks on scattering for nonlinear Schrödinger equations*, NODEA Nonlinear Differential Equations Appl., **9** (2002), 45–68, doi:10.1007/s00030-002-8118-9.

[21] T. OZAWA AND K. YAMAUCHI , *Remarks on analytic smoothing effect for the Schrödinger equation*, Math. Z., **261** (2009), 511–524, doi:10.1007/s00209-008-0334-5.

[22] T. OZAWA AND K. YAMAUCHI, *Analytic smoothing effect for global solutions to nonlinear Schrödinger equation*, J. Math. Anal Appl., **364** (2010), 492–497, doi:10.1016/j.jmaa.2009.09.033.

[23] T. OZAWA, K. YAMAUCHI AND Y. YAMAZAKI, *Analytic smoothing effect for solutions to Schrödinger equations with nonlinearity of integral type* Osaka Math. J., **42** (2005), 737–750.

[24] L. ROBBIANO AND C. ZUILY, *Microlocal analytic smoothing effect for the Schrödinger wquation*, Duke Math. J., **100** (1999), 93–129.

[25] L. ROBBIANO AND C. ZUILY, *Effect régularisant microlocal analytique pour l'equation de Schrödinger : le cas données oscillantes*, Comm. Partial Deifferential Equations, **25** (2000), 1891–1906.

[26] R.S. STRICHARTZ, *Restrictions of Fourier transforms to quadratic surfaces and decay of solutions of wave equations*, Duke. Math. J., **44** (1977), 705–714, doi:10.1215/S0012-7094-77-04430-1.

[27] J.C.H SIMON AND E. TAFLIN, *Wave operators and analytic solutions of nonlinear Klein-Gordon equations and of nonlinear Schrödinger equations*, Commun. Math. Phys., **99** (1985), 541–562, doi:10.1007/BF01215909.

[28] C. SULEM AND P.-L. SULEM, *The nonlinear Schrödinger equation. Self-focusing and wave collapse*, Appl. Math. Sci., **139**, Springer 1999.

[29] H. TAKUWA, *Analytic smoothing effects for a class of dispersive equations*, Tsukuba J. Math., **28** (2004), 1–34.

[30] K. YAJIMA, *Existence of solutions for Schrödinger evolution equations*, Commun. Math.Phys., **110** (1987), 415–426, doi:10.1007/BF01212420.

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