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RESONANT BOUNDARY VALUE PROBLEMS FOR SINGULAR MULTI–TERM FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this article, we present and study three class of resonant boundary value problems for singular fractional differential equations with sup-multiplicative-like operators. The existence results for solutions of these boundary-value problems are established. Our analysis relies on the well known coincidence degree theory. Here the nonlinearity terms in fractional differential equations depend on $D^{\gamma}_{0+}u$ and may be singular at $t=0$ or $t=1$. The results obtained generalize and enrich known results to some extent from the literature.

Mathematics subject classification (2010): 92D25, 34A08, 34B16, 47H10.
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