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SOLOW DIFFERENTIAL EQUATIONS ON TIME SCALES – A UNIFIED APPROACH TO CONTINUOUS AND DISCRETE SOLOW GROWTH MODEL

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Abstract. In this paper we reformulate the axioms of the well-known Solow macroeconomic growth model by means of the mathematical calculus on time scales. We derive a system of differential equations on a time scale $\mathbb{T}$ which is a generalization of the classical Solow fundamental differential equation for the continuous case as well as its discrete version. We also prove sufficient conditions for the exponential stability of equilibrium points of this system having positive coordinates. Applications of these results to the case of the Cobb-Douglas production function are given.

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