

RANDOM SET-VALUED FUNCTIONAL DIFFERENTIAL EQUATIONS WITH THE SECOND TYPE HUKUHARA DERIVATIVE

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Abstract. This paper concerns with the initial value problem for random set-valued functional differential equations with the second type Hukuhara derivative (RSFDEs). By using the techniques of successive approximations, the existence and uniqueness of solutions are established. Two kinds of boundedness of the solution are also established. In addition, the problem at least one solution under some conditions is proven and two examples illustrate the results.

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