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OSCILLATORITY OF FRESNEL INTEGRALS AND CHIRP–LIKE FUNCTIONS

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Abstract. In this review article, we present results concerning fractal analysis of Fresnel and generalized Fresnel integrals. The study is related to computation of box dimension and Minkowski content of spirals defined parametrically by Fresnel integrals, as well as computation of box dimension of the graph of reflected component function which are chirp-like function. Also, we present some results about relationship between oscillatority of the graph of solution of differential equation, and oscillatority of a trajectory of the corresponding system in the phase space. We are concentrated on a class of differential equations with chirp-like solutions, and also spiral behavior in the phase space.

Mathematics subject classification (2010): 28A12, 28A75, 37C45, 34C15.
Keywords and phrases: box dimension, oscillations, Fresnel integral, Bessel function, clothoid, chirp, spiral, wavy spiral.

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