Differential
Equations
& Applications
Volume 5, Number 4 (2013), 561–576
doi:10.7153/dea-05-33

EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR SEMILINEAR ELLIPTIC SYSTEMS INVOLVING HARDY–SOBOLEV CRITICAL NONLINEARITY

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Abstract. This paper is concerned with a singular elliptic system, which involves the Hardy-Sobolev critical nonlinearity. The existence and multiplicity of solutions for this system are obtained by the variational methods.

Mathematics subject classification (2010): 35A15, 35B33, 35J70.
Keywords and phrases: solution, critical Hardy-Sobolev exponent, elliptic system.

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Paper DEA-05-33