

## EXISTENCE AND CONCENTRATION OF GROUND STATE SOLUTION TO A CRITICAL $p$ -LAPLACIAN EQUATION

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**Abstract.** In this paper, we consider the existence and concentration behavior of positive ground state solution to the following problem

$$\begin{cases} -h^p \Delta_p u + V(x)|u|^{p-2}u = K(x)|u|^{q-2}u + |u|^{p^*-2}u, & x \in \mathbb{R}^N, \\ u \in W^{1,p}(\mathbb{R}^N), u > 0, & x \in \mathbb{R}^N, \end{cases}$$

where  $h$  is a small positive parameter,  $1 < p < N$ ,  $\max\{p, p^* - \frac{p}{p-1}\} < q < p^*$ ,  $p^* = \frac{Np}{N-p}$  is the critical Sobolev exponent,  $V(x)$  and  $K(x)$  are positive smooth functions. Under some necessary restrictions, we show that for small  $h > 0$ , the equation has a positive ground state solution. Furthermore, we establish the concentration property of such solutions when  $h$  tends to zero.

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