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EXISTENCE RESULT OF POSITIVE SOLUTION FOR BOUNDARY VALUE PROBLEMS OF FRACTIONAL ORDER WITH INTEGRO–DIFFERENTIAL BOUNDARY CONDITIONS

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Abstract. In this paper we study the following fractional boundary value problem with integro-differential boundary conditions

$$\begin{cases} D^{\alpha}_{0^+}u(t) - f(t,u(t),D^{\alpha-1}_{0^+}u(t),D^{1-\alpha}_{0^+}u(t)) = 0,\ t\in[0,T],\ n-1\leqslant\alpha<n, \\ u^{(j)}(0) = 0,\ D^{\alpha-1}_{0^+}u(T) + \int_0^T u(\omega)d\omega + \sum_{i=1}^{m-2}\beta_i u(\xi_i) = 0,\ j=0,\ldots,n-2, \\ 0<\xi_i<\xi_{i+1}<T,\ \beta_i\in[0,\infty),\ i=1,2,\ldots,m-2,\ n\in\mathbb{N}\setminus\{1\},\ T>0, \end{cases}$$

where $D^{\alpha}_{0^+}, D^{\alpha-1}_{0^+}$ represent the standard Riemann-Liouville fractional derivative of order α. The main result includes some interesting fixed point and functional analysis techniques to obtain claimed existence result.

Mathematics subject classification (2010): fractional derivative, generalized boundary conditions, positive solutions, fixed point theory.

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REFERENCES

[1] B. AHMAD, S.K. NTOUYAS, *Nonlinear fractional differential equations and inclusions of arbitrary order and multi-strip boundary conditions*, Electronic Journal of Differential Equations, **98**, Vol. 2012 (2012), 1–22.

[2] A. CABADA, G. WANG, *Positive solutions of nonlinear fractional differential equations with integral boundary value conditions*, Journal of Mathematical Analysis and Applications, **389**, (2012), 403–411.

[3] K. GHANBARI, Y. GHOLAMI, *Existence and multiplicity of positive solutions for m-point nonlinear fractional differential equations on the half line*, Electronic Journal of Differential Equations, **238**, Vol. 2012 (2012), 1–15.

[4] K. GHANBARI, Y. GHOLAMI, *Existence and nonexistence results of positive solution for nonlinear fractional eigenvalue problem*, Journal of Fractional Calculus and Applications, **2**, Vol. 4. Jan. 2013, 1–12.

[5] K. GHANBARI, Y. GHOLAMI, H. MIRZAEI, *Existence and multiplicity results of positive solutions for boundary value problems of nonlinear fractional differential equations* , Dynamics of Continuous, Discrete and Impulsive Systems Series **A**: Mathematical Analysis, **20**, (2013), 543–558.

[6] L.P. LEBEDEV, I.I. VOROVICH, G.M.L. GLADWELL, *Functional Analysis (Applications in Mechanics and Inverse Problems, 2^{end} Edition)*, Kluwer Academic Publishers, **2002**.

[7] A.A. KILBAS, H.M. SRIVASTAVA, J.J. TRUJILLO, *Theory and Applications of fractional Differential Equations*, North-Holland mathematics studies,Elsvier science, **204**, (2006).

[8] N. KOSMATOV, *Solutions to a class of nonlinear differential equations of fractional order*, Electronic Journal of Qualitative Theory of Differential Equations, **20**, (2009), 1–13.

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[9] S.K. NTOUYAS, *Boundary value problems for nonlinear fractional differential equations and inclusions with nonlocal and fractional integral boundary conditions*, Opuscula Math, **33**, (2013), 117–138.
[10] I. PODLUBNY, *Fractional Differential Equations*, Mathematics in Science and Applications, Academic Press, **19** (1999).
[11] G. WANG, S. LIU, T.P. AGARWAL, L. ZHANG, *Positive solutions of integral boundary value problem involving Riemann-Liouville fractional derivative*, Journal of Fractional Calculus and Applications, **4** (2) (2013), 312–321.
[12] S. ZHANG, *Positive solutions for boundary value problem of nonlinear fractional differential equations*, Ellectronic Journal of Differential Equations, **36**, Vol. 2006(2006), 1–12.
[13] X. SU, S. ZHANG, *Unbounded solutions to a boundary value problem of fractional order on the half-line*, Computers and Mathematics with Applications, **61** (2011), 1079–1087.
[14] X. ZHAO, W. GE, *Unbounded solutions for a fractional boundary value problem on the infinite interval*, Acta Appl Math, **109** (2010), 495–505.
[15] W. ZHONG, W. LIN, *Nonlocal and multiple-point boundary value problem for fractional differential equations*, Comput. Math. Appl, **39** (2010), 1345–1351.