

ON THE UNIQUENESS OF WEAK SOLUTIONS FOR THE 3D PHASE FIELD NAVIER–STOKES VESICLE–FLUID INTERACTION MODEL

JIHONG ZHAO, QIAO LIU AND SHUANGHU ZHANG

Abstract. In this paper, we study a hydrodynamical system modeling the deformation of vesicle membrane in incompressible viscous fluids. In three dimensional case, we establish some uniqueness criteria of weak solutions for this system which reveal that the regularity of velocity field alone controls the uniqueness of weak solutions.

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