A LEWY–STAMPACCHIA INEQUALITY IN VARIABLE SOBOLEV SPACES FOR PSEUDOMONOTONE OPERATORS

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Abstract. In this paper, we are interested in proving the Lewy-Stampacchia inequality in the general framework of an obstacle problem for a nonlinear pseudomonotone elliptic operator in $W_0^{1,p(\cdot)}(\Omega)$ where $p(\cdot)$ is a log-Hölder continuous exponent. Our aim is to adapt to the context of variable exponent Sobolev spaces a previous work of the first author, based on a penalization method.

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