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NONTRIVIAL SOLUTIONS FOR SYSTEMS OF STURM–LIOUVILLE BOUNDARY VALUE PROBLEMS

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Abstract. Sufficient conditions are established for the existence of at least one nontrivial classical solution to the boundary value system with Sturm-Liouville boundary conditions

$$\begin{cases} -(\phi_{p_i}(u_i'(x)))' = \lambda F_{u_i}(x,u_1,\dots,u_n)h_i(u_i'(x)) & \text{in } (a,b), \\ \alpha_i u_i(a) - \beta_i u_i'(a) = 0, \quad \gamma_i u_i(b) + \sigma_i u_i'(b) = 0, \end{cases} \qquad i = 1,\dots,n.$$

The analysis is based on variational methods and critical point theory.

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