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BLOWUP OF NONLINEAR SCHRÖDINGER EQUATIONS WITH INVERSE–SQUARE POTENTIALS

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Abstract. Blowup in finite time for nonlinear Schrödinger equations (NLS) with inverse-square potential $a|x|^{-2}$ and nonlocal nonlinearities described by integral operators $\mathbf{(HE)}_a$ is considered. The local and global existence for $\mathbf{(HE)}_a$ is studied in Suzuki [10]. To show the blowup for (NLS) the virial identity is important role. But the identity for $\mathbf{(HE)}_a$ has not proved in consequence of the strongly singular potential. Thus we give a strict proof of the virial identity for $\mathbf{(HE)}_a$.

Mathematics subject classification (2010): 35Q55, 35Q40, 81Q15.
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