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BIFURCATION TYPE PHENOMENA FOR POSITIVE SOLUTIONS OF NONLINEAR NEUMANN EIGENVALUE PROBLEMS

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Abstract. We consider a parametric nonlinear problem driven by the p-Laplace differential operator and with a reaction which is p-superinear near $+\infty$ but need not satisfy the usual in such cases Ambrosetti-Rabinowitz condition. Using critical point theory and truncation and comparison techniques, we prove a bifurcation-type theorem for such problems.

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