

ON ENTIRE SOLUTIONS FOR AN INDEFINITE QUASILINEAR SYSTEM OF MIXED POWER

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Abstract. We prove non-existence and existence of entire positive solutions for a Schrödinger quasilinear elliptic system. To prove the non-existence, we combine a carefully-chosen test function with some results that we proved concerning the positivity of a kind of principal eigenvalue of an eigenvalue problem in \mathbb{R}^N with indefinite weights. Contrary to the existence, the non-existence results for this class of problems have not been studied very much in recent years. For the existence we mainly used upper and lower solution methods combined with comparison principles.

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