Differential
Equations
& Applications
Volume 6, Number 3 (2014), 403–415
doi:10.7153/dea-06-22

EXISTENCE AND ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS FOR A CLASS OF $(p(x),q(x))$– LAPLACIAN SYSTEMS

HONGHUI YIN AND ZUODONG YANG

Abstract. In this paper, our main purpose is to establish the existence of positive solution of the following system

$$\begin{cases} -\Delta_{p(x)}u = u^{\alpha(x)} + \lambda^{p(x)}v^{m(x)}, & x \in \Omega \\ -\Delta_{q(x)}v = v^{\beta(x)} + \theta^{q(x)}u^{n(x)}, & x \in \Omega \\ u = v = 0, \quad x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with C^2 boundary, $p(x), q(x)$ are functions which satisfy some conditions, $-\Delta_{p(x)}u = -\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u)$ is called $p(x)$-Laplacian. We give the existence results of positive solutions and consider the asymptotic behavior of the solutions near the boundary. The approach is based on the sub- and super-solution method.

Mathematics subject classification (2010): 35J60, 35J62.
Keywords and phrases: positive solution, $(p(x),q(x))$-Laplacian, asymptotic behavior, sub-supersolution.

REFERENCES

[1] E. Acerbi, G. Mingione, Regularity results for a class of functionals with nonstandard growth, Arch. Ration. Mech. Anal., **156** (2001), 121–140.
[2] E. Acerbi, G. Mingione, Regularity results for stationary electro-rheological fluids, Arch. Ration. Mech. Anal., **164** (2002), 213–259.
[3] C.H. Chen, On positive weak solutions for a class of quasilinear elliptic systems, Nonlinear Anal., **62** (2005), 751–756.
[4] X.L. Fan, Y.Z. Zhao, Q.H. Zhang, A strong maximum principle for $p(x)$-Laplacian equations, Chinese J. Contemp. Math., **24**, 3 (2003), 277–282.
[5] X.L. Fan, Global $C^{1,\alpha}$ regularity for variable exponent elliptic equations in divergence form, J. Differential Equations, **235** (2007), 397–417.
[6] X.L. Fan ,D. Zhao, On the spaces $L^{p(x)}(\Omega)$ and $W^{m,p(x)}(\Omega)$, J. Math. Anal. Appl., **263** (2001), 424–446.
[7] X.L. Fan, D. Zhao, The quasi-minimizer of integral functionals with $m(x)$ growth conditions, Nonlinear Anal., **39** (2000), 807–816.
[8] X.L. Fan, H.Q. Wu, F.Z. Wang, Hartman-type results for $p(t)$-Laplacian systems, Non-linear Anal., **52** (2003), 585–594.
[9] X.L. Fan, Y.Z. Zhao, D. Zhao, Compact imbedding theorems with symmetry of Strauss-Lions type for the spaces $W^{1,p(x)}$, J. Math. Anal. Appl., **255** (2001), 333–348.
[10] X.L. Fan, Q.H. Zhang, Existence of solutions for $p(x)$-Laplacian Dirichlet problem, Nonlinear Anal., **52** (2003), 1843–1852.
[11] X.L. Fan, On the sub-supersolution method for $p(x)$-Laplacian equations, J. Math. Anal. Appl., **330** (2007), 665–682.
[12] D. Gilbarg, N.S. Trudinger, Elliptic Partial Differential Equations of Second Order, Springer-Verlag, Berlin, 1998.
[13] D.D. Hai, R. Shivaji, An existence result on positive solutions of p-Laplacian systems, Nonlinear Anal., **56** (2004), 1007–1010.

Paper DEA-06-22

[14] A. El Hamidi, Existence results to elliptic systems with nonstandard growth con-ditions, J. Math. Anal. Appl., **300** (2004), 30–42.
[15] O. Kováčik,J.Rákosník, On the spaces $L^{p(x)}(\Omega)$ and $W^{k,p(x)}(\Omega)$, Czechoslovak Math. J., **41** (1991), 592–618.
[16] M.Rúzicka, Electro-rheological Fluids: Modeling and Mathematical Theory, Lecture Notes in Math., vol.1784, Springer-Verlag, Berlin, 2000.
[17] S.G.Samko, Denseness of $C_0^\infty(\mathbb{R}^N)$ in the generalized Sobolev spaces $W^{m,p(x)}(\mathbb{R}^N)$, Dokl. Ross. Akad. Nauk, **369**, 4 (1999), 451–454.
[18] H.H. Yin, Z.D. Yang, Existence and nonexistence of entire positive solutions for quasilinear systems with singular and super-linear terms, Differ. Equ. Appl., **2** (2010), 241–249.
[19] Q.H. Zhang, Existence of positive solutions for a class of $p(x)$-Laplacian systems, J.Math. Anal. Appl., **333** (2007), 591–603.
[20] Q.H. Zhang, Existence of positive solutions for elliptic systems with nonstandard $p(x)$-growth conditions via sub-supersolution method, Nonlinear Anal., **67** (2007), 1055–1067.
[21] Q.H. Zhang, Existence and asymptotic behavior of positive solutions for a variable exponent elliptic system without variational structure, Nonlinear Anal., **72** (2010), 354–363.
[22] Q.H.Zhang, Existence of radial solutions for $p(x)$-Laplacian equations in $\mathbb{R}^N$, J. Math. Anal. Appl., **315**, 2 (2006), 506–516.
[23] V.V. Zhikov, Averaging of functionals of the calculus of variations and elasticity theory, Math. USSR Izv., **29** (1987), 33–36.