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STABILITY RESULTS OF SOME ABSTRACT EVOLUTION EQUATIONS

N.S. HOANG

Abstract. The stability of the solution to the equation $\dot{u} = A(t)u + G(t,u) + f(t)$, $t \geqslant 0$, $u(0) = u_0$ is studied. Here $A(t)$ is a linear operator in a Hilbert space H and $G(t,u)$ is a nonlinear operator in H for any fixed $t \geqslant 0$. We assume that $\|G(t,u)\| \leqslant \alpha(t)\|u\|^p$, $p > 1$, and the spectrum of $A(t)$ lies in the half-plane $\mathrm{Re}\,\lambda \leqslant \gamma(t)$ where $\gamma(t)$ can take positive and negative values. We proved that the equilibrium solution $u(t) \equiv 0$ to the equation is Lyapunov stable under persistantly acting perturbations $f(t)$ if $\sup_{t\geqslant 0}\int_0^t \gamma(\xi)\,d\xi < \infty$ and $\int_0^\infty \alpha(\xi)\,d\xi < \infty$. In addition, if $\int_0^t \gamma(\xi)\,d\xi \to -\infty$ as $t \to \infty$, then we proved that the equilibrium solution $u(t) \equiv 0$ is asymptotically stable under persistantly acting perturbations $f(t)$. Sufficient conditions for the solution $u(t)$ to be bounded and for $\lim_{t\to\infty} u(t) = 0$ are proposed and justified.

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