Differential
Equations
& Applications
Volume 6, Number 4 (2014), 467–480
doi:10.7153/dea-06-27

A DELAY VIRUS MODEL WITH BEDDINGTON–DEANGELIS FUNCTIONAL RESPONSE

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Abstract. In this paper, we present a delay virus model with Beddington-DeAngelis functional response. We first introduce the basic reproduction number R_0 and the immune response reproduction number R_1, and then show that the system has three possible equilibria depended on R_0 and R_1. We further show that the global stability of the disease-free equilibrium E_0, immune-free equilibrium E_1 and endemic equilibrium E_2 are fully determined by R_0 and R_1, that is, E_0, E_1 and E_2 are globally asymptotically stable when $R_0 \leqslant 1$, $R_1 \leqslant 1 < R_0$, and $R_1 > 1$, respectively.

Mathematics subject classification (2010): 94O25, 92C50, 34O03.
Keywords and phrases: delay virus model, global stability; Beddington-DeAngelis functional response, Lyapunov functionals, LaSalle invariance principle.

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