

A DELAY VIRUS MODEL WITH BEDDINGTON–DEANGELIS FUNCTIONAL RESPONSE

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Abstract. In this paper, we present a delay virus model with Beddington-DeAngelis functional response. We first introduce the basic reproduction number R_0 and the immune response reproduction number R_1 , and then show that the system has three possible equilibria depended on R_0 and R_1 . We further show that the global stability of the disease-free equilibrium E_0 , immune-free equilibrium E_1 and endemic equilibrium E_2 are fully determined by R_0 and R_1 , that is, E_0 , E_1 and E_2 are globally asymptotically stable when $R_0 \leq 1$, $R_1 \leq 1 < R_0$, and $R_1 > 1$, respectively.

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