EXISTENCE OF SOLUTION FOR BIHARMONIC
SYSTEMS WITH INDEFINITE WEIGHTS

GAURAV DWIVEDI

Abstract. In this article we deal with the existence questions to the nonlinear biharmonic systems. Using theory of monotone operators, we show the existence of a unique weak solution to the weighted biharmonic systems. We also show the existence of a positive solution to weighted biharmonic systems in the unit ball in $\mathbb{R}^n$, using Leray Schauder fixed point theorem. In this study we allow sign-changing weights.

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