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ASYMPTOTIC AND OSCILLATORY BEHAVIOR OF nTH–ORDER HALF–LINEAR DYNAMIC EQUATIONS

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Abstract. In this paper, we study the nth-order half-linear dynamic equations

$$(x^{[n-1]})^{\Delta}(t) + p(t)\,\phi_{\alpha[1,n-1]}(x(g(t))) = 0$$

on an above-unbounded time scale $\mathbb{T}$, where $n \geqslant 2$,

$$x^{[i]}(t) := r_i(t)\phi_{\alpha_i}\left[\left(x^{[i-1]}\right)^{\Delta}(t)\right],\ i = 1,\dots,n-1,\ \text{ with } x^{[0]} = x,$$

$\phi_\beta(u) := |u|^\beta \operatorname{sgn} u$, and $\alpha[i,j] := \alpha_i \cdots \alpha_j$. Criteria are obtained for the asymptotics and oscillation of solutions for both even and odd order cases. This work extends several known results in the literature on second-order, third-order, and higher-order linear and half-linear dynamic equations.

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