

SINGLE-POINT BLOW-UP FOR A SEMILINEAR REACTION-DIFFUSION SYSTEM

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Abstract. In this paper, we consider positive solutions of the system

$$u_t - \Delta u = u^r v^p, \quad v_t - \Delta v = u^q v^s$$

$t \in (0, T)$, $x \in \mathbf{B}(0, R) = \{x \in \mathbb{R}^n \mid |x| < R\}$ or $x \in \mathbb{R}^n$ and $p, q, r, s > 1$. We prove single-point blow-up if $r < q + 1$ and $s < p + 1$ and for a large class of radial decreasing solutions. This extends the result of Friedman and Giga for this basic system known only for $p = q = r = s$. We also obtain lower pointwise estimates for the blow-up profiles.

Mathematics subject classification (2010): 35B20, 35B40, 35B50, 35K55, 35K57, 35K58.

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